Minimum Spanning Tree : Prim's Algorithm

Prepared by – Subhra Sarkar

Assistant Professor, BSH Dept.

Bengal Institute of Technology

Module V – Course Contents

- Basic concept of graph
- Walk, Path, Circuit
- Euler and Hamiltonian graph
- Digraph
- Matrix representation: Incidence and Adjacency matrix

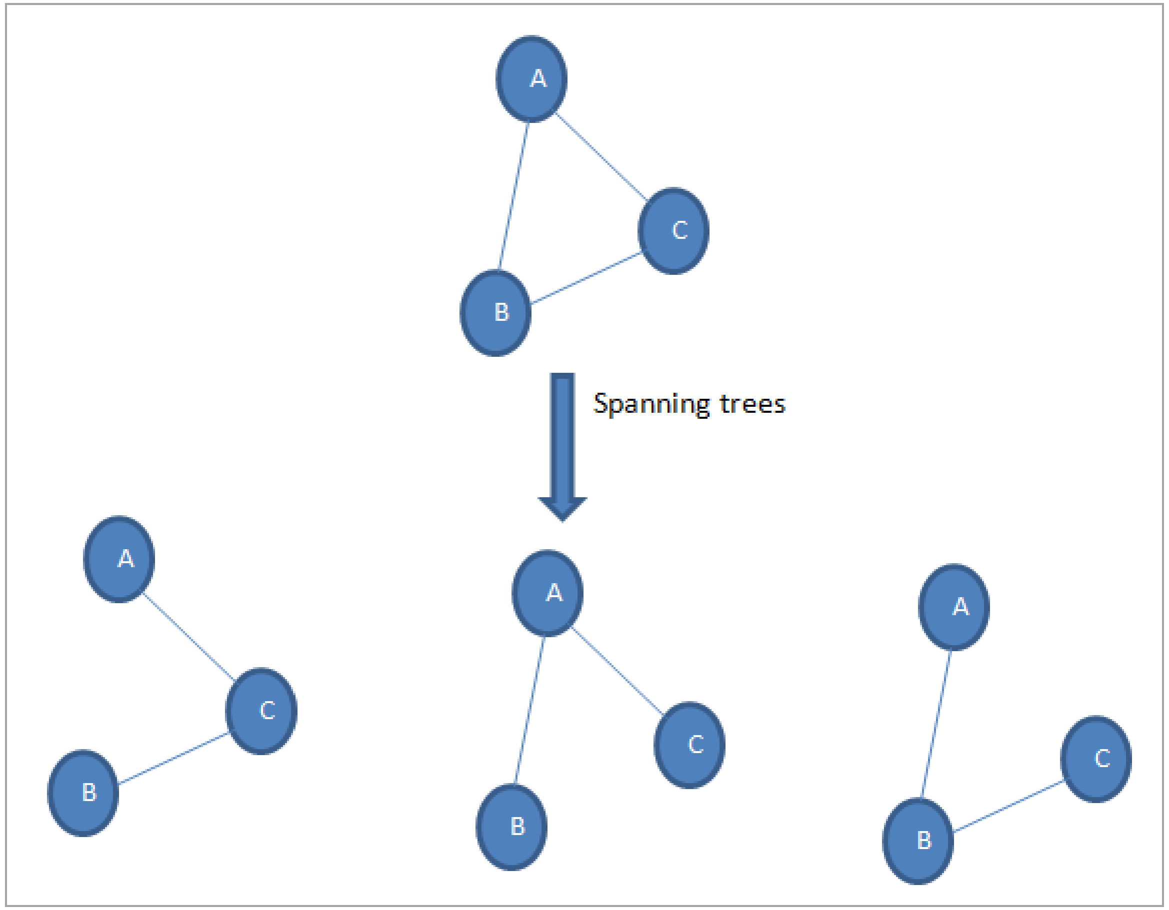
- **Tree: Basic concept of tree**
- Binary tree
- Spanning tree
- Kruskal and Prim's algorithm for finding the MST
- Dijkstra's Algorithm for finding the Shortest Path between nodes

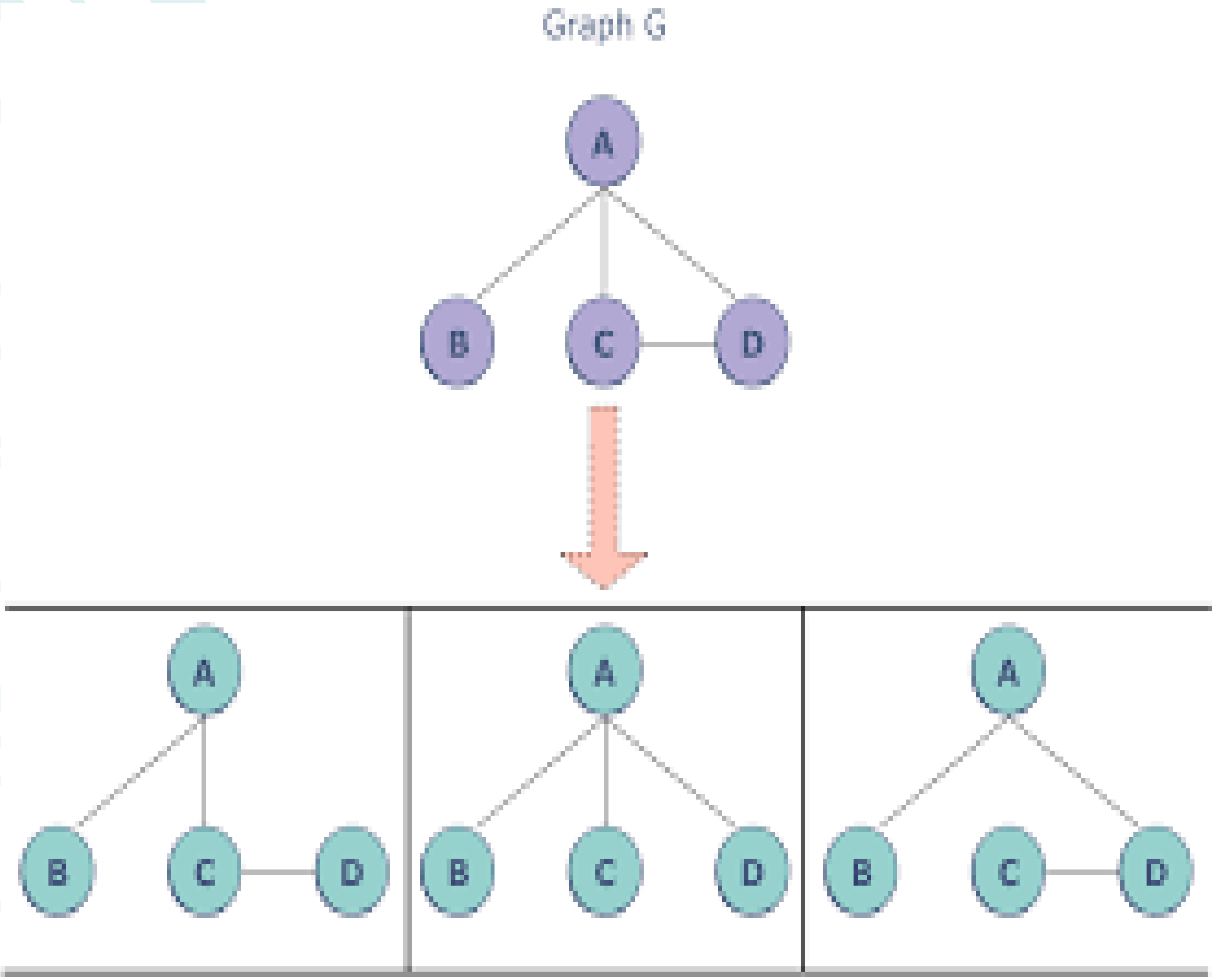
Tree
Connected & Acyclic Graph

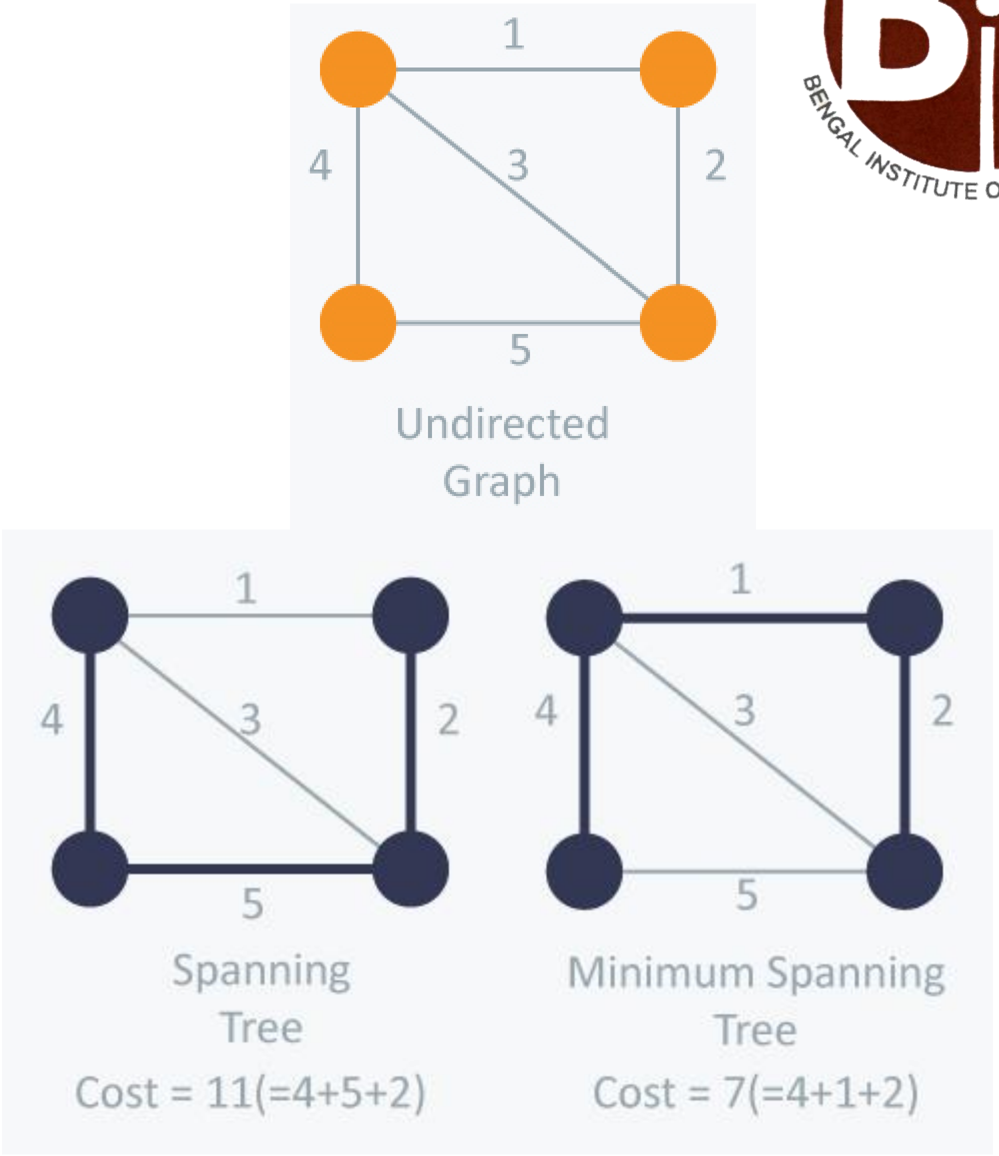
- A connected acyclic (circuit free) graph is known as Tree.

  ✔ No loop
  ✔ No parallel edges

❑ **Spanning Tree -** Given an undirected and connected graph G = (V,E), a spanning tree of the graph G is a tree that spans G (that is, it includes every vertex of G) and is a subgraph of G (every edge in the tree belongs to G).




Graph G
A
B
C
D

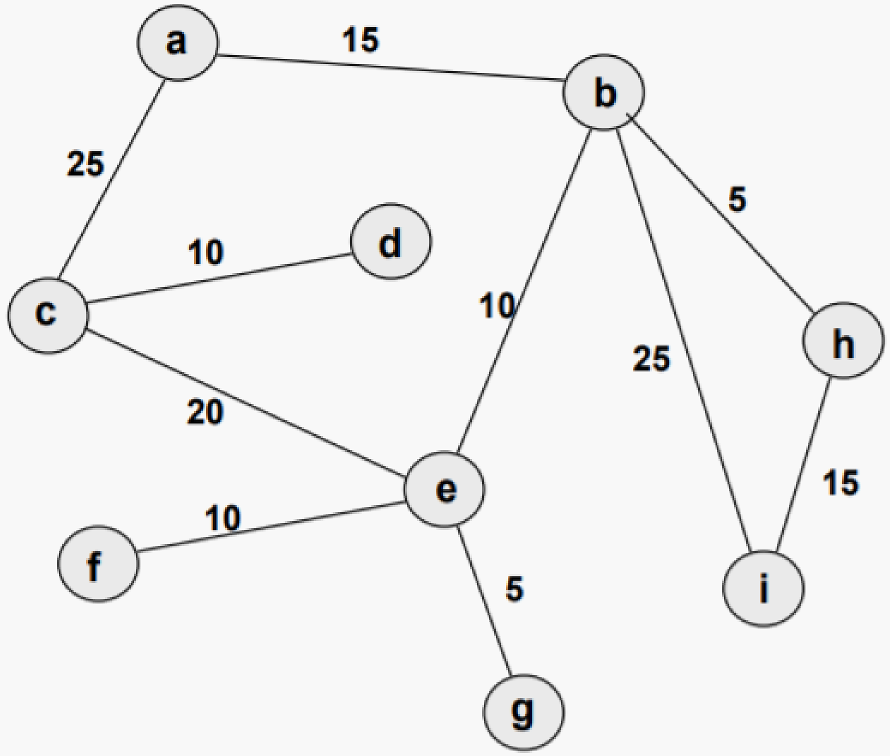

1
4
3
2
5
Undirected Graph
Spanning Tree
Cost = 11(=4+5+2)
Minimum Spanning Tree
Cost = 7(=4+1+2)

❑ Weighted Graph -

In many applications, each edge of a graph has an associated numerical value, called a <u>weight</u>.

Usually, the edge weights are non-negative integers.

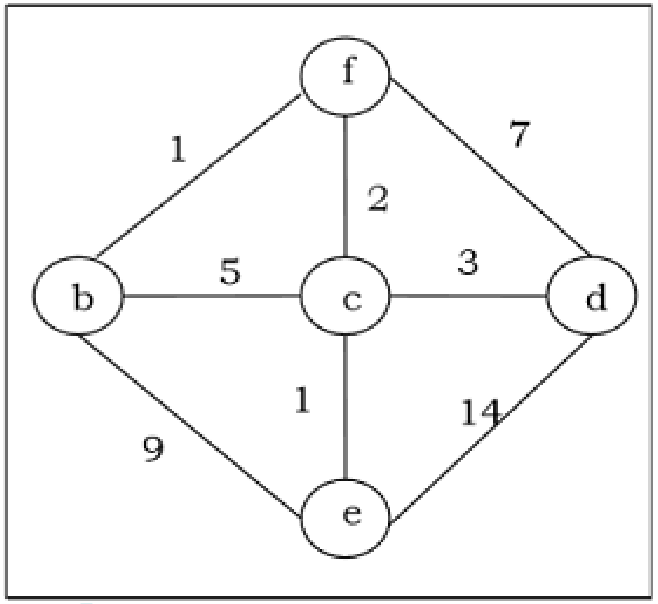
Weighted graphs may be either directed or undirected.

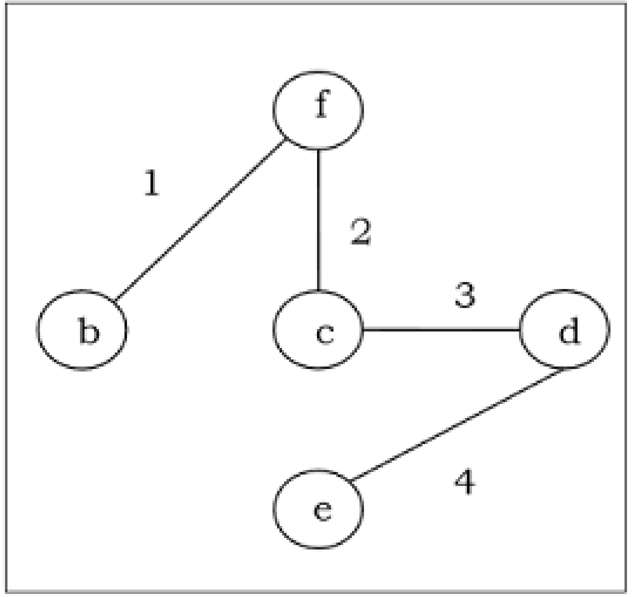


The weight of an edge is often referred to as the "cost" of the edge.

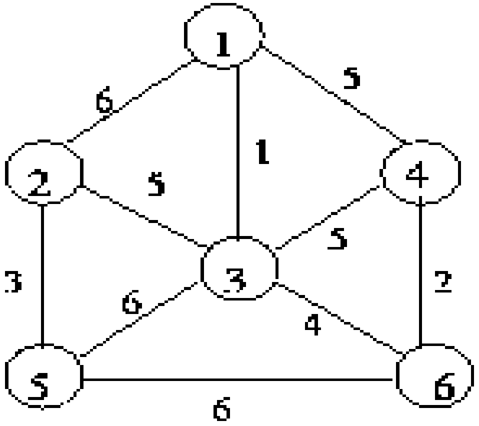
In applications, the weight may be a measure of the length of a route, the capacity of a line, the energy required to move between locations along a route, etc.

- **Minimum Spanning Tree -** The cost of the spanning tree is the sum of the weights of all the edges in the tree. There can be many spanning trees. Minimum spanning tree is the spanning tree where the cost is minimum among all the spanning trees.

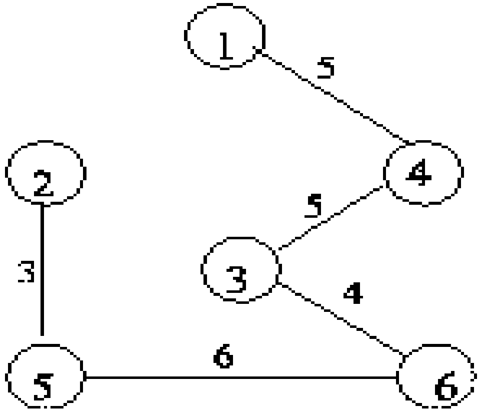




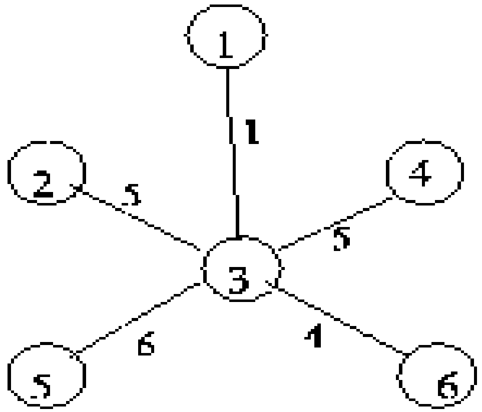
Prepared by - Subhra Sarkar, Basic Science & Humanities Department, Bengal Institute of Technology, Email - subhrasarkar.bit@gmail.com



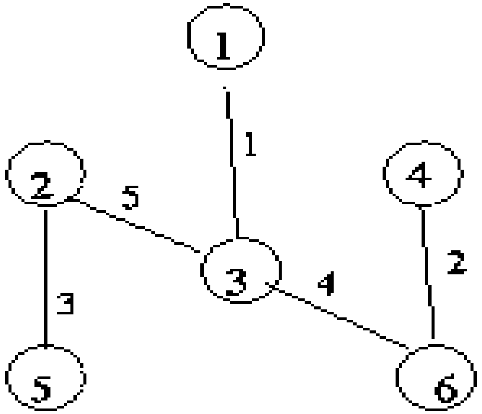
A connected graph.



A spanning tree with cost = 23



Another spanning tree with cost 21



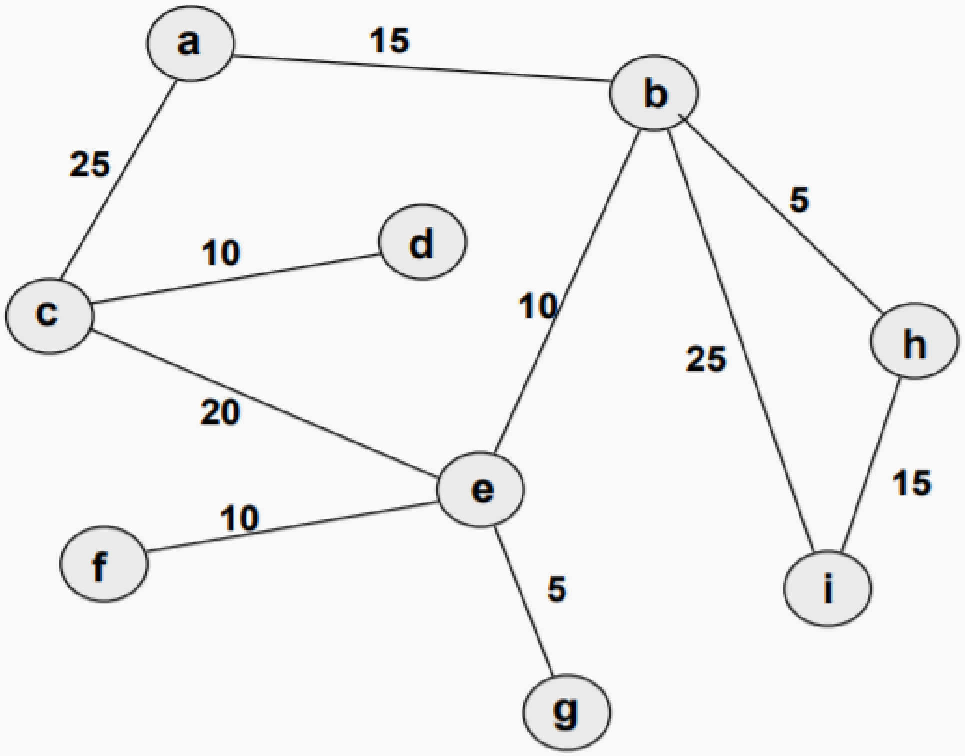
MST, cost = 15

Minimal Spanning Tree

Given a weighted graph, we would like to find a spanning tree for the graph that has minimal total weight.

The total weight of a spanning tree is the sum of the weights of its edges.

We want to find a spanning tree T, such that if T' is any other spanning tree for the graph then the total weight of T is less than or equal to that of T'.



❑ Prim's Algorithm -

- Prim's Algorithm is a famous greedy algorithm.
- It is used for finding the Minimum Spanning Tree (MST) of a given graph.
- To apply Prim's algorithm, the given graph must be weighted, connected and undirected.

▪ **Step I -**

- Randomly choose any vertex.
- The vertex connecting to the edge having least weight is usually selected.

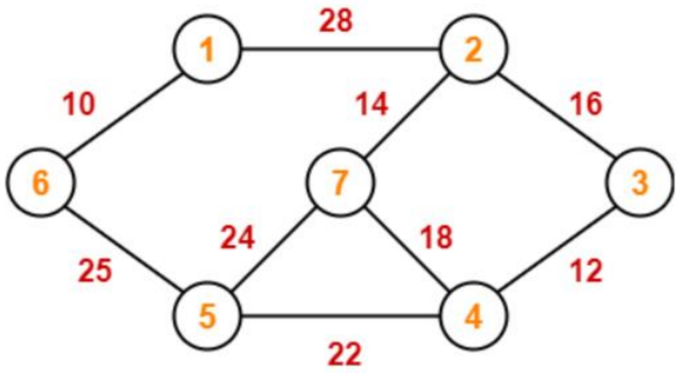
▪ **Step II -**

- Find all the edges that connect the tree to new vertices.
- Find the least weight edge among those edges and include it in the existing tree.
- If including that edge creates a cycle, then reject that edge and look for the next least weight edge.

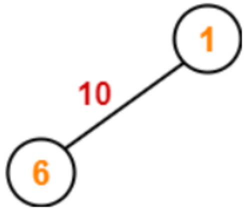
▪ **Step III -**

- Keep repeating step-II until all the vertices are included and Minimum Spanning Tree (MST) is obtained.

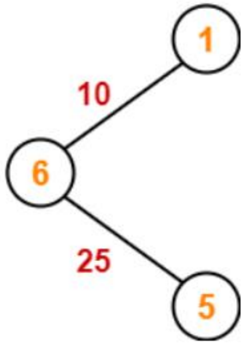
❑ Example -



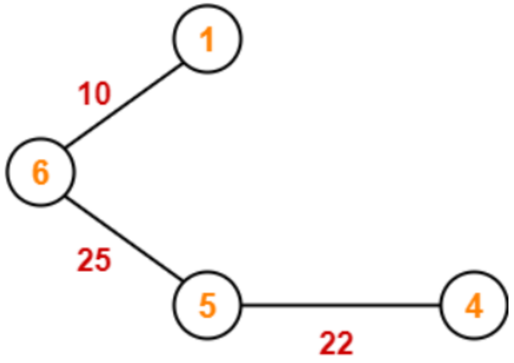
- **Step I -**



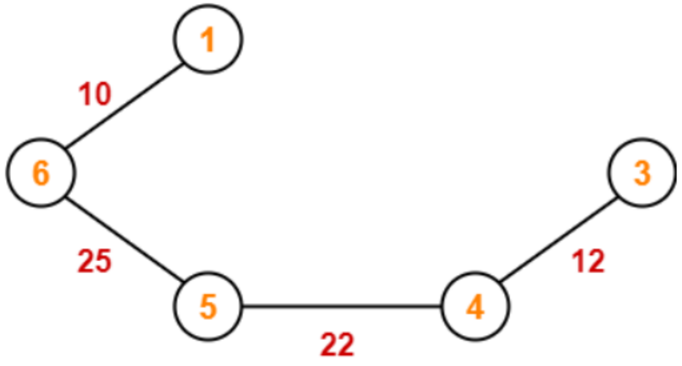
- **Step II -**



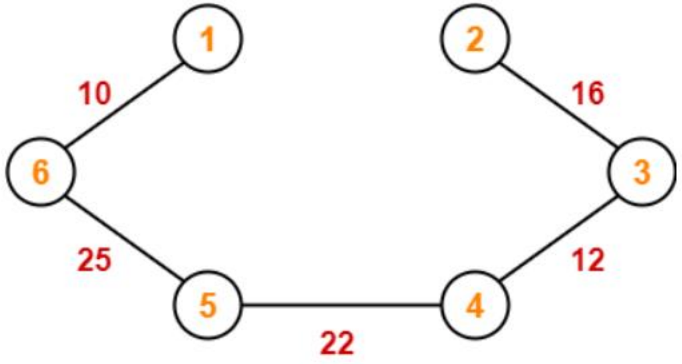
- **Step III -**



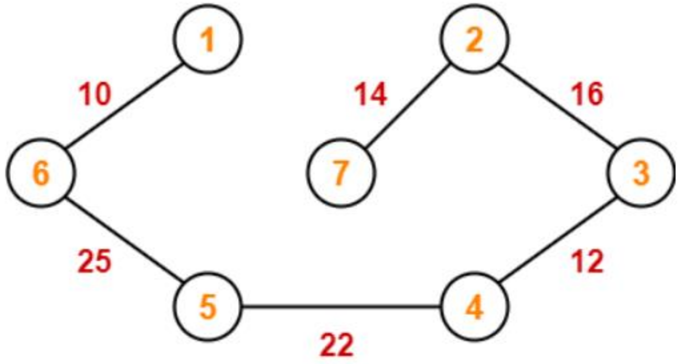
- **Step IV -**

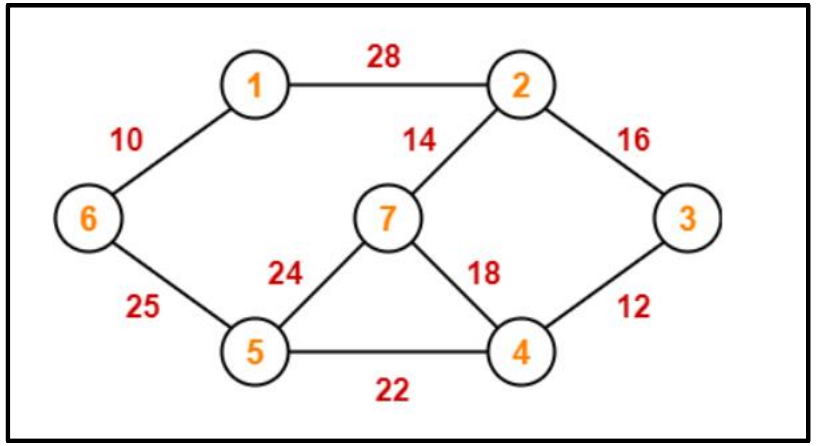


- **Step V -**



- **Step VI -**

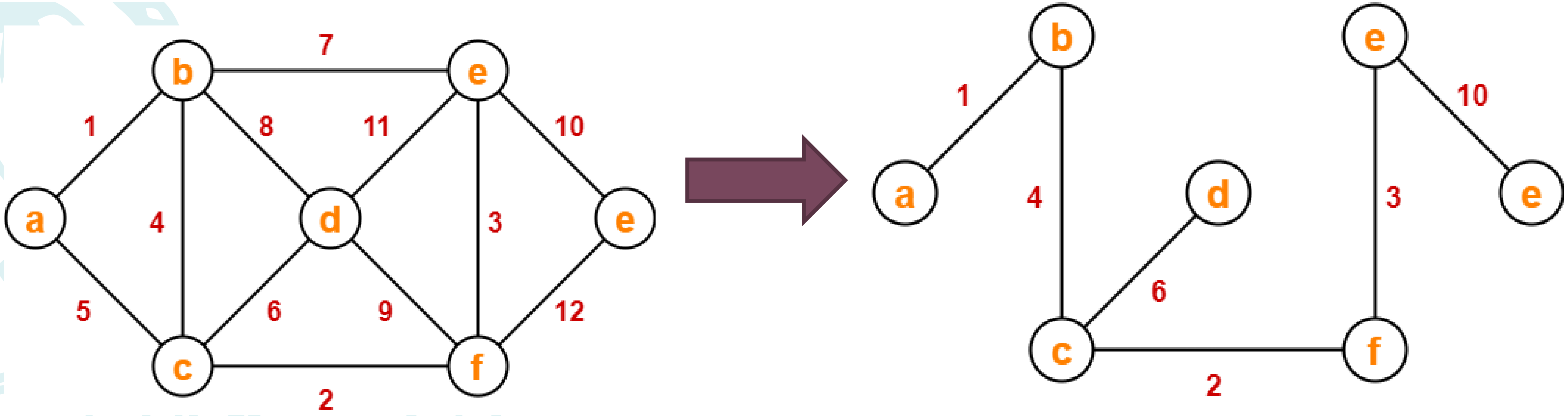




- **Since all the vertices have been included in the MST, so we stop.**

- **Now, Cost of Minimum Spanning Tree = Sum of all edge weights**
  = 10 + 25 + 22 + 12 + 16 + 14
  = 99 units

❑ Example -



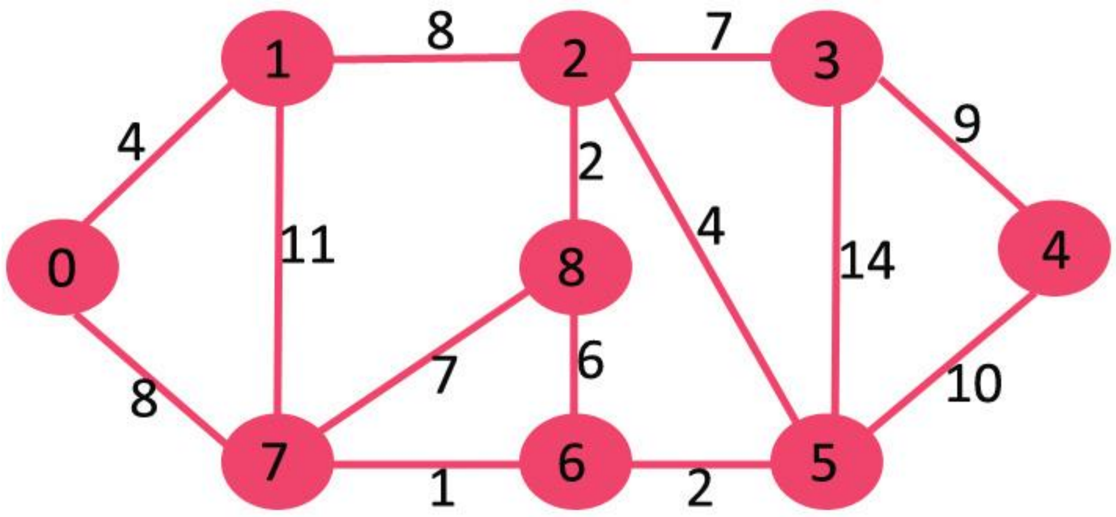
Now, Cost of Minimum Spanning Tree = Sum of all edge weights

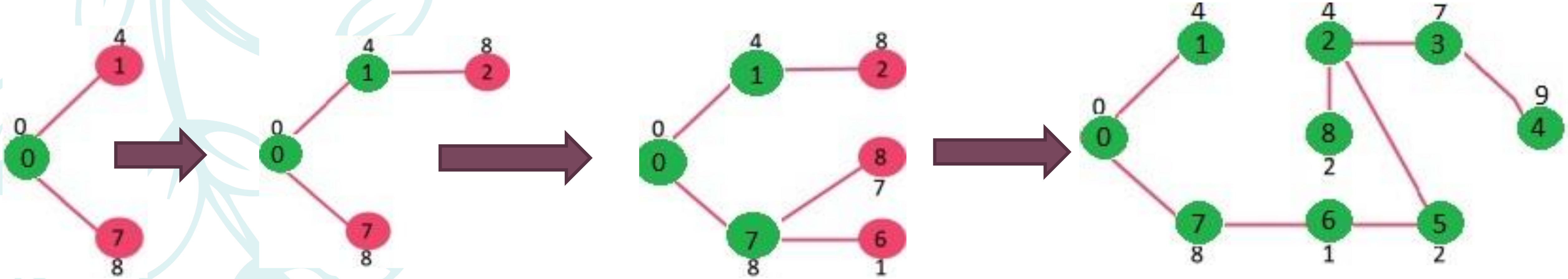
= 1 + 4 + 2 + 6 + 3 + 10

= 26 units

Prepared by - Subhra Sarkar, Basic Science & Humanities Department, Bengal Institute of Technology, Email - subhrasarkar.bit@gmail.com

❑ Example -





Now, Cost of Minimum Spanning Tree = Sum of all edge weights

= 4+8+1+2+2+4+7+9

= 37 units

Prepared by - Subhra Sarkar, Basic Science & Humanities Department, Bengal Institute of Technology, Email - subhrasarkar.bit@gmail.com

Thank You