Minimum Spanning Tree : Kruskal's Algorithm

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- **Spanning Tree -** Given a connected and undirected graph, a spanning tree of that graph is a subgraph that is a tree and connects all the vertices together.

✓ A single graph can have many different spanning trees.

- **Minimum Spanning Tree -** A minimum spanning tree (MST) or minimum weight spanning tree for a weighted, connected and undirected graph is a spanning tree with weight less than or equal to the weight of every other spanning tree. The weight of a spanning tree is the sum of weights given to each edge of the spanning tree..

Kruskal’s Algorithm -

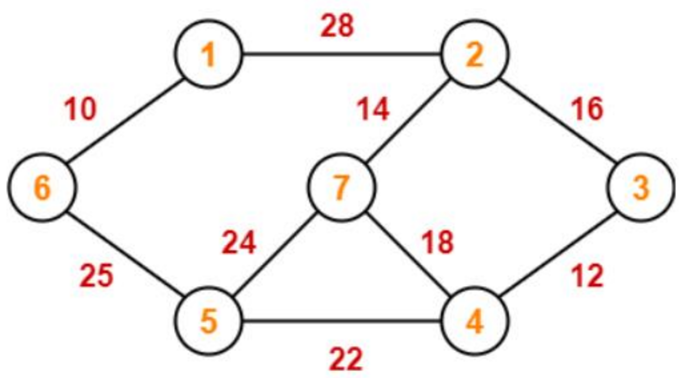
- Kruskal’s Algorithm is a famous greedy algorithm.
- It is used for finding the Minimum Spanning Tree (MST) of a given graph.
- To apply Kruskal’s algorithm, the given graph must be weighted, connected and undirected.

- **Step I -**
  - Sort all the edges in non-decreasing order of their weight.

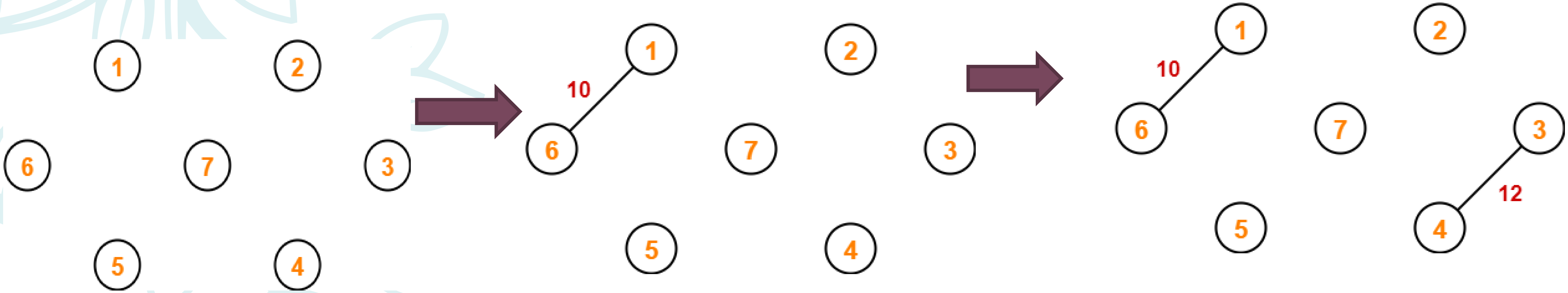
- **Step II -**
  - Take the edge with the lowest weight and use it to connect the vertices of graph.
  - If adding an edge creates a cycle, then reject that edge and go for the next least weight edge.

- **Step III -**
  - Keep adding edges until all the vertices are connected and a Minimum Spanning Tree (MST) is obtained.

❑ Example -



- ✓ Simply draw all the vertices on the paper.
- ✓ Connect these vertices using edges with minimum weights such that no cycle gets formed.



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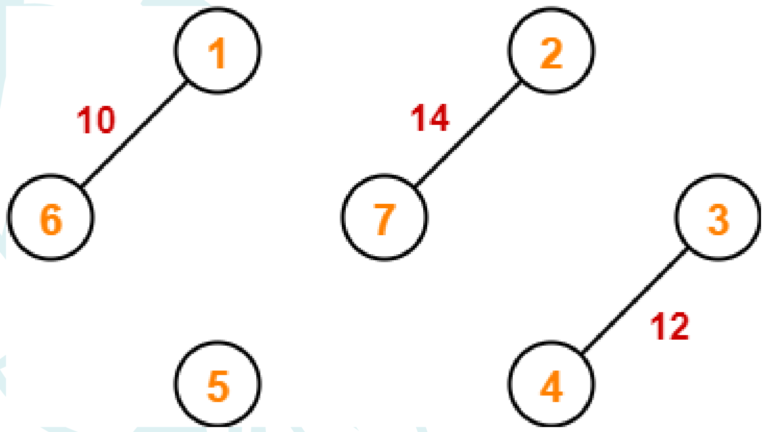
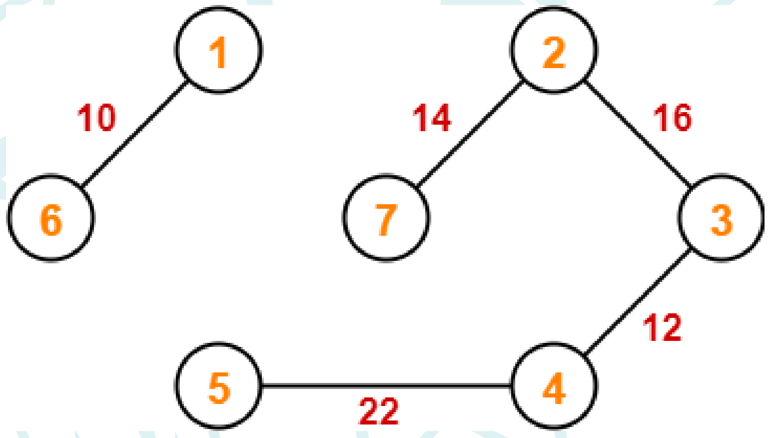
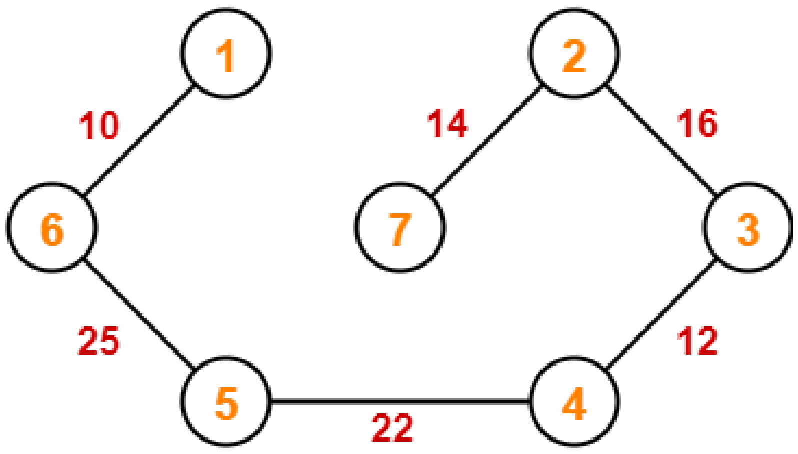




- **Since all the vertices have been included in the MST, so we stop.**
- **Now, Cost of Minimum Spanning Tree**
  = Sum of all edge weights
  = 10 + 25 + 22 + 12 + 16 + 14
  = 99 units



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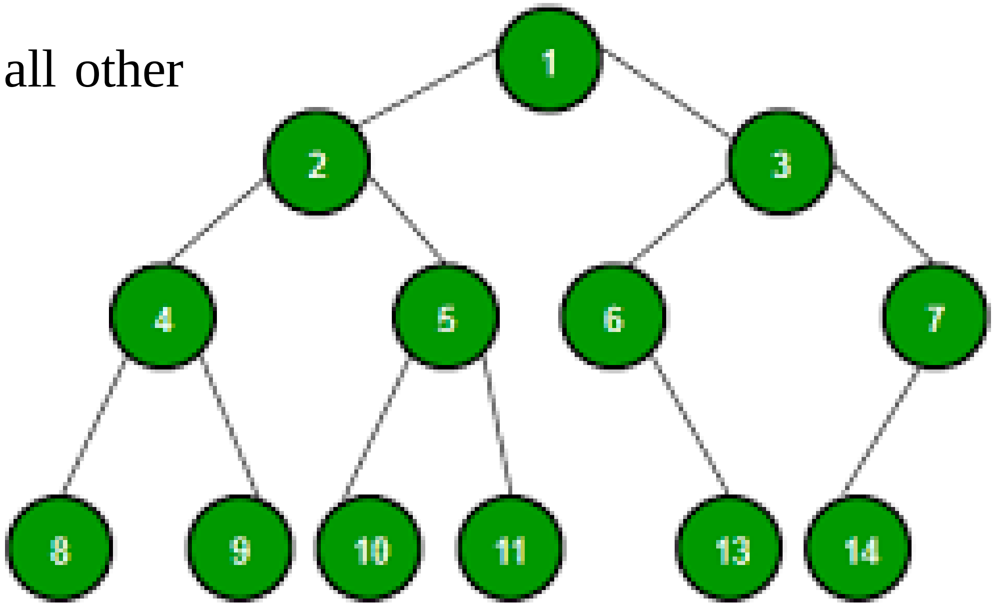
- **Binary Tree -** A binary tree is a tree-like structure that is rooted and in which each vertex has at most two children and each child of a vertex is designated as its left or right child.

- In a binary tree there is only one vertex of degree two and all other vertices are of degree either one or three.



- The vertex of degree two present in this tree is called root node.
- All other vertices are of degree one or three.
- One degree vertices are the pendant nodes and remaining vertices are internal vertex.

- Result – The number of vertices in a binary tree is always odd.

Ans – Since any binary tree has one vertex of degree two and all other vertices are of degree 1 and 3, so in this case the number of odd degree vertices is n-1. Again we know that the number of odd vertices in a graph is even. So n – 1 = even, i.e. n = odd.

- Result – The number of pendent vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree.

Ans – Let k be the number of pendant vertices in a binary tree. Also the tree being a binary tree, it has some vertices of degree 3. So the number of vertices of degree 3 is n – x – 1. Thus the sum of all degrees = 1.k + 2.1 +3.(n-k-1) = 3n -2k -1.

But, we know that the sum of degrees of the vertices in a graph is twice the number of the edge in the graph. So

$$3n - 2k - 1 = 2(n-1), \qquad i.e.\ 2k = n+1, \qquad i.e.\ k = \frac{n+1}{2}$$

So the number of pendant vertices is $\frac{n+1}{2}$.

- Result – The number of internal vertices in a binary tree is one less than the number of pendant vertices.

Ans – In a tree, a vertex which is not a pendant vertex of the tree is called a non – pendant or internal vertex. Let in a binary tree, number of pendant vertices are k and those internal vertices are m. So if n be the number of all vertices, then $n = k + m$.

By the previous result, the number of pendant vertices $k = \frac{n+1}{2}$.

So, number of internal vertices $m = n - k = n - \frac{n+1}{2} = \frac{n-1}{2} = \frac{n+1}{2} - 1 = k - 1$

Thus the number of internal vertices in a binary tree is one less than the number of pendant vertices.

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Thank You