Important Questions from Sequence, Series, Multivariate, Differential Equation

Mathematics - III

1. Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$

2. Evaluate $\int_0^a \int_0^{\sqrt{(a2-y2)}} (x^2 + y^2)\, dy\, dx$ changing to polar co-ordinate. (Ans - $\pi a^4/8$)

3. Find the volume common to the cylinder $x^2 + y^2 = 9$ and $x^2 + z^2 = 9$. (Ans-144)

4. Show that the vector $A = (6xy+z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrortational. Find a scalar function f such that $A = \nabla f$. (Ans - f = 3x2y + xz3 - yz + c)

5. Evaluate $\int_0^a \int_0^x \int_0^y x^3y^2z\, dz\, dy\, dx$ (Ans - $a^9 / 90$)

6. Find the equation of the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point (2, -1, 5). (Ans - 4x-2y-z = 5, (x-2)/4 = (y+1)/-2 = (z-5)/-1)

7. Find the area of the triangle whose vertices are (1, 3), (0, 0), (1, 0). (Ans - 3/2)

8. Find grad f where $f = 3x^2y - y^3z^2$ at the point (1, -2, -1). (Ans - -12i - 9j - 16k)

9. Find the maximum value of x^3y^2 subject to the constraint $x + y = 1$ using the method of Lagrange's multiplier.

10. Find the Jacobian of u, v, w with respect to x, y, z where u = yz/x, v = zx/y, w = xy/z

11. Examine the existence of maxima or, minima, if any, of the function $f(x, y) = x^2 + y^2 + (x + y + 1)^2$

12. If $f(x, y) = \log e\, (x^2 + y^2)/ (x + y)$, prove that $x\, \partial f/\partial x + y\, \partial f/\partial y = 1$

13. Calculate $\partial(u, v)/\partial(x, y)$ where $u = 2xy$, $v = x^2 - y^2$

14. If $u = \log_e r$ and $r^2 = x^2 + y^2 + z^2$, prove that $r^2(\partial^2u/\partial x^2)+(\partial^2u/\partial y^2)+(\partial^2u/\partial z^2)=1$
15. If f(x, y) = (x-y)/(x+y), find f_x and f_y at (2, -1) from the definition.

16. Solve $x^2\, d^2y/dx^2 + x\, dy/dx + y = \log_e x \sin(\log_e x)$

17. Find the P.I. of $(D^2 + 2)y = x^2$

18. Solve $x^2\, dy/dx + xy = y^2$

19. Find the general singular solution of $y = 4xp - 16y^3p^2$

20. Solve by the method of variation of parameters, $d^2y/dx^2 + a^2y = \sec ax$

21. Solve (x+y+1)dy/dx = 1

22. Solve $(D^2 - 5D + 5)y = x^2 e^{3x}$

23. Find the wronskian of two function $y_1 = \sin ax$, $y_2 = - \cos ax$

24. Find the general solution and singular solution of $yp^2 – 2xp + y = 0$, where p = dy/dx

25. Reduce the equation $(p-1)\, e^{3x} + p^3\, e^{2y} = 0$ into the Clairaut's form.

26. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. (Ans - $4a^3 / 3$)

27. Evaluate $\iint \sqrt{(4x^2-y^2)}\, dx\, dy$ over the traiangle formed by the straight lines $y = 0$, $x = 1$, $y = x$. (Ans - $\sqrt{3} / 6 + \pi/9$)

28. Find div grad f, where $f = 2x^2y^3z^4$. (Ans - $4yz^2(y^2z^2 + 3x^2z^2 + 6x^2y^2)$)

29. Prove that $f(x, y) = xy^2 / x^2 + y^4, x \neq 0$

$= 0, x = 0$

is not differentiable at (0, 0) though it possess first order partial derivatives at (0, 0)

30. Verify Euler's Theorem for the function $u = (x-y)/(x+y)$

31. Find the saddle points of the function $x^3 + y^3 - 63x -63y + 12xy$

32. Solve $y^2 \log_e y = xyp + p^2$

33. Find a particular integral of $d^2y/dx^2 + y = 3/2$

34. Verify that $y = xe^{3x}$ is a solution of $d^2y/dx^2 – 4dy/dx + 3y = 2e^{3x}$

35. Solve $(d^2y/dx^2)+4\, dy/dx+4y=0, y(0)=1, (dy/dx)x=0=0$

36. Solve $y = 2px + p^4x^2$

37. Solve $(y-px)(p-1) = p$ and obtain the singular solution.

38. Discuss the convergent of the series $1/2 + 1.3/2.4 + 1.3.5/2.4.6 + \cdots$ (Ans - Use Raabe's test, D' Alembert's ratio test will fail here)

39. Examine the convergence of the series $\sum ((n/(n+1))n)2$. (Ans - Use cauchy root test)

40. Examine the series for convergence $1.2/2 + 2.2/2.2 + 3.2/2.3 + 4.2/2.4 +\cdots$ (use D'Alemberts ratio Test)

41. Show that the sequence{xn}, where $xn=1+ 1/3+ 1/3^2 + 1/3^3 +\cdots$ converges. Find its limit. (Ans - limit = 3/2)