Practice Set on Graph Theory -

1. Is there any simple graph corresponding to the following degree sequences?
   a. (1, 1, 2, 3)
   b. (2, 2, 4, 6)
   c. (1, 3, 3, 4, 5, 6, 6)
2. Draw simple graphs, if possible, with the following properties. Give explanation where the graph is not possible.
   a. 5 vertices each of degree 4
   b. 5 vertices each of degree 2
   c. 6 vertices having degree 5, 5, 4, 2, 1, 1
   d. 5 vertices having degree 1,2,2,4,5
   e. 20 edges if each vertex is of degree 4
3. Find the maximum number of vertices in a connected graph having 45 edges.
4. Find the minimum number of edges in a connected graph having 15 vertices.
5. Show that the number of odd degree vertices in a simple graph is always even.
6. Prove that a simple graph with $n \geq 2$ vertices has at least two vertices of same degree.
7. Show that maximum number of edges in a simple graph with n vertices is .
8. Draw a graph, if exists, having the following properties or explain why no such graph exists: A graph with 4 edges, 4 vertices with degree sequence 1,2,3,4.
9. Prove that the minimum number of edges in a connected graph with n vertices is n-1.
10. If a simple regular graph has n vertices and 24 edges, find all possible values of n. Draw a graph against each of such values of n.
11. Suppose G is an undirected graph with 12 edges. If G has 6 vertices, each of degree 3 and rest have degree less than 3, then find the minimum number of vertices in G.
12. a. Find the no of edges in a complete graph with 6 vertices.

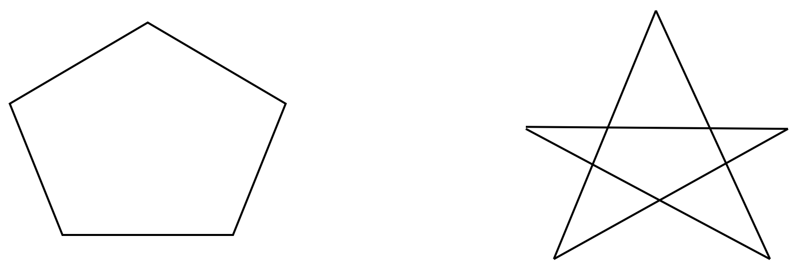
    b. Find the maximum number of edges of a simple graph with 8 vertices and 4 components.
13. Prove that a simple graph with n number of vertices must be connected if it has more than (n-1)(n-2)/2 edges.
14. If G be a simple graph having at most 2m vertices and if the degree of every vertex of G is greater than or equal to m, then show that G is connected.
15. If in a connected graph, having at least two vertices, the number of edges isless than the number of vertices, then the graph has a pendant vertex.
16. Show that there exists no simple graph having 3 vertices of degree 3 each and one vertexof degree 1.
17. Let G be a graph having p no of vertices and q no of edges such that all the vertices havedegree k or (k + 1). If G has t > 0 vertices of degree k, then show that t = p (k+1) - 2q.

18. Show whether the following graphs are isomorphic or not:

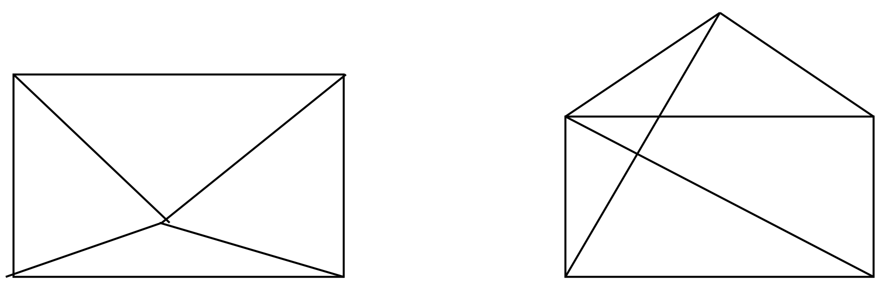
a)

b)

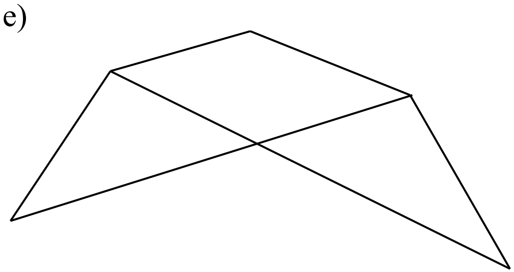
c)



d)



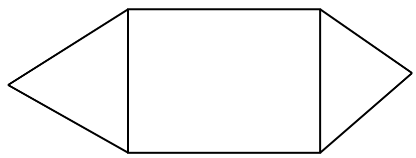
e)



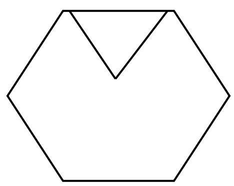
f)

19. Define complement of a graph. Draw the complement for the following graphs.

a)

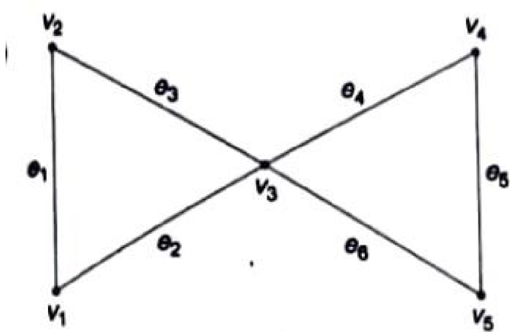


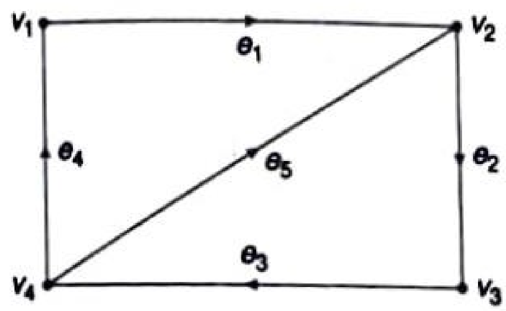
b)



20. If G be a simple graph with n vertices and e edges then how many edges does the compliment of G have?

21. Find the incidence matrix of the following graph/di-graph.

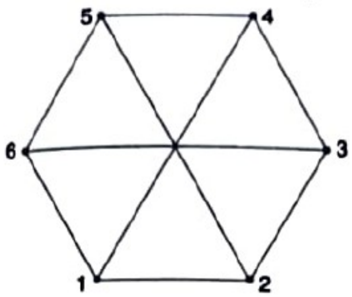


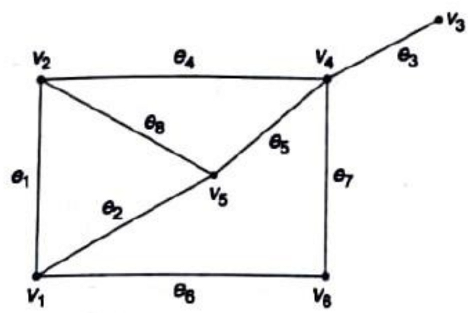


22. Draw the graphs of the following incidence matrices.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

23. Find the adjacency matrix of the following graph/digraph.





24. Draw the graph of the following adjacency matrices.

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	0	1	0	0	0
v_2	0	0	0	0	1	0
v_3	0	0	0	1	0	0
v_4	0	0	0	1	1	0
v_5	0	0	0	0	0	1
v_6	0	1	0	0	0	1

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	1	1	1
v_2	0	0	1	1	1
v_3	1	1	0	0	0
v_4	1	1	0	0	0
v_5	1	1	0	0	0

25. Define Euler circuit. Write the necessary and sufficient condition for a graph to contain an Eulerian circuit. Find, if possible, an Euler circuit in the following graph: